



Economic and  
Social Commission  
for Western Asia

Optimization model  
for poverty reduction  
strategies

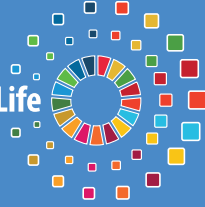


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Committed to the 2030 Agenda, ESCWA's passionate team produces innovative knowledge, fosters regional consensus and delivers transformational policy advice. Together, we work for a sustainable future for all.

Economic and Social Commission for Western Asia

# Optimization model for poverty reduction strategies



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## KEY MESSAGES



01

Successful recovery from the COVID-19 pandemic and implementation of the 2030 Agenda for Sustainable Development require strong policy responses.



02

The optimization model developed by the Economic and Social Commission for Western Asia (ESCWA) facilitates the optimized allocation of resources across population groups and multidimensional poverty index (MPI) indicators.



03

Using survey information on trends in household deprivation and data from the ESCWA Social Expenditure Monitor (SEM), the optimization model aligns the record of deprivations with the projected resources required for their mitigation with a view to proposing efficient interventions.

# Executive summary

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Beneficiary identification and support allocation is a well-known challenge in the implementation of social protection programmes, particularly when those interventions take several different forms. Interventions can be implemented by a wide range of stakeholders, including governments, national private sector donors, international donors, religious institutions and households. The MPI expands traditional measures of financial poverty and captures deprivations in multiple dimensions related to individuals' capabilities and well-being. The MPI is a useful tool for determining the distribution of multiple deprivations across population groups, including by incorporating geographic and demographic differences. To date, however, MPI research and applications have not provided national planners with relevant instruments to ensure the efficient use of limited resources and the achievement of poverty reduction targets.

The study outlined in the present report is an initial attempt to address those challenges. Two optimization models have been developed to that end, each with its own strengths, weaknesses, assumptions and results. Each model is characterized either by the type of information available to the policymaker, or by political and technological restrictions for targeting interventions, and specifically whether measuring and targeting can be conducted at the household level or through geographic or demographic assessments. A complete mathematical formulation for each of the two optimization models has been developed.

The two models are tested against sample data from Lebanon. We discuss the process involved and the performance of the two models and highlight how the results can support decision makers in identifying the interventions that are most effective at meeting MPI reduction targets and the demographic cells that should be prioritized.

The household-level targeting scenario is conceptual by design, as no State is assumed to have the necessary information or the political and technological capacity to put that scenario into practice. Demographic cell-level targeting, however, is a realistic scenario that can be used to improve efficiency provided that the State can target a large number of geographic cells. It directs policymakers to explore differences among geographic cells, thus enabling them to spot mismatches between resource allocation and poverty measures.

A custom illustration using survey data from Lebanon confirms that informed household-level interventions can achieve poverty reduction targets and require less effort than limited targeting at the government level. Indeed, targeting at the national level may sometimes yield inferior outcomes. However, examples of limited information governorate-level targeting show results comparable to those of informed household-level targeting.

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# Introduction

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The Multidimensional Poverty Index (MPI) is a measure of poverty that takes into consideration several dimensions beyond traditional monetary poverty. It provides insight into the main sources of socioeconomic deprivation and combines poverty headcount and poverty intensity into one robust measure. The inclusion of various dimensions, including education, health, living standards and employment informs stakeholders regarding the extent and the nature of multidimensional poverty in terms of aspects that go beyond material wealth (Alkire and Santos, 2014; Sen, 1976; Tsui, 2002).

The MPI was developed by the United Nations Development Programme (UNDP) in collaboration with the Oxford Poverty and Human Development Initiative (OPHI). Multidimensional poverty is a useful indicator when measuring fundamental deprivations at the global level, while still capturing the specificities of each region. It can also be used to delve into greater detail and measure deprivations at the country level. The conventional headcount ratio used to compute the percentage of people living below the poverty line, which is typically calculated in relation to the cost of meeting basic needs, is adjusted to encompass multiple dimensions other than income poverty. The headcount ratio is therefore adjusted to aggregate information on deprivation according to several indicators to create a deprivation score, identify who is poor using that score, and then aggregate across sampled households to calculate estimates across the population. The global MPI includes ten indicators and has a fully outlined methodology (Alkire and Foster, 2011); (Alkire and Santos, 2014).

The approach of the MPI methodology relies on the use of data collated in demographic health surveys on household income, expenditure and consumption surveys or any other surveys that capture multiple indicators of wealth attainment or deprivation. The Economic and Social Commission for Western Asia (ESCWA) has adapted the MPI framework and approach to measure the nature of Arab multidimensional poverty.

ESCWA has also collaborated with OPHI to measure the MPI at the regional level, and has developed specific national frameworks for many countries in the Arab region. At the regional level, the pioneering Arab MPI framework was developed by ESCWA and its partners (ESCWA and others, 2017), and the recently revised version (ESCWA and others, 2020) was formally endorsed by the League of Arab States Social Ministerial Council for use in the forthcoming 2022 Arab Multidimensional Poverty Report.

Identifying and measuring the impact that immediate, medium and long-term shocks have on multidimensional poverty is key for policy advancement, design and evaluation. For the purpose of this study, shocks are defined as events leading to large shifts in deprivation that have measurable effects on households. Shocks can be internal and defined as societal interventions, such as reforms of social protection policies aimed at providing assistance to those in need, or external, including events beyond the control of society. Any shock can affect the lives of people across society, albeit to varying degrees. A complete assessment can be conducted more accurately if the shocks have already had their full impact and micro-level survey data are available.

If the shocks have not yet had an impact, or if sufficient data on conditions before and after the shock are unavailable, assessments may need to be undertaken using microsimulation techniques in order to understand the impact on deprivation in terms of individual indicators and on overlapping deprivations at the household level. Alternative shock scenarios or probable outcomes may also be considered. Simulations rely on several assumptions regarding how shock impacts are distributed and propagated across indicators and households over time, and are influenced by interlinkages among indicators and the knock-on effect this has on households.

In countries that have experienced an economic crisis, and where the more recent surveys only consider data prior to the shock, modellers quantifying the change in deprivation of households using the MPI approach will try to answer the following questions: How many households and which ones will change status from non-deprived to deprived per indicator? Will this overlap with household deprivations according to other indicators and change the status of households to multidimensionally poor? How will impacts propagate over time? And how will demographic change interact with the impacts between the pre- and post-shock periods?

Despite their reliance on assumptions, microsimulations are crucial to policymakers for computing deprivation dashboard indicators and MPIs, and planning contingency responses to a range of possibilities and the multifaceted nature of shocks.

One way to capture the direction of change at the indicator level is to adopt a joint distribution copula modelling approach, by identifying external factors influencing the dynamics of selected indicators, measuring relationships using historical data and generating a multidimensional poverty index from the counterfactual projection on the basis of all indicators (ESCWA, 2021a). Other proposed models now consider deprivation at the dimension level and concentrate on selected dimensions such as health (Klassen and Lange, 2012). By focusing on specific initiatives implemented by governments, such as school closures, or on food insecurity estimates using measures such as the United Nations Children's Fund (UNICEF) tracker, World Bank Education simulations, World Bank Economic Outlooks or UNDP forecasts, one can forecast the short-term impact of the COVID-19 pandemic on multidimensional poverty under various scenarios (Alkire and others, 2021; UNICEF, 2022).

In contrast to existing projection and simulation literature, the present report aims to support policymakers in efficiently reducing multidimensional poverty in a targeted manner, keeping feasibility and historical relevance in mind. The outcomes can facilitate an assessment of whether government resource allocation plans are in line with poverty reduction requirements.

The remainder of the present report is organized as follows: Chapter 2 outlines the theoretical modelling framework, Chapter 3 presents the mathematical formulation used, while Chapter 4 provides an overview of the two alternative simulations and their outcomes, highlighting key findings. Chapter 5 reviews the limitations of current assessment methods and outlines areas that would lend themselves to future analysis. Chapter 6 sets out the conclusions reached in the study.

# 1. Proposed models

---

The MPI identifies multiple deprivations at the household and individual levels using several indicators grouped into health, education and standard of living categories (UNDP, 2013). The computation of the MPI relies on microdata from household surveys, meaning the indicators needed to construct such measures must come from the same surveys. A deprivation cut-off is set for each indicator so that all individuals can be identified as deprived or not deprived with respect to each indicator. The MPI framework labels a household as poor if its combined deprivations across indicators exceed the deprivation cut-off. National MPI measures can be derived based on this bottom-up approach.

The MPI is composed of a set of poverty measures with the headcount or incidence of multidimensional poverty equivalent to the proportion of people whose MPI falls above the predetermined MPI cut-off for multidimensional poverty. The intensity of multidimensional poverty is the weighted average of the number of deprivations a poor person might experience simultaneously. The MPI score consolidates information regarding the incidence and intensity of multiple deprivations into a single number. The score is calculated by multiplying the multidimensional poverty headcount by the intensity of poverty. These measures can be analysed to illustrate the composition of multidimensional poverty both across countries and regions and within countries by ethnic group, urban/rural area, age and other key household and community characteristics.

The MPI methodology demonstrates how multidimensionally poor people are deprived and helps reveal interdependencies among those deprivations. That insight enables policymakers to funnel resources and design policies more effectively. The MPI can also be analysed to show disaggregation across various sociodemographic and geographic cells, revealing, for example, which areas or groups are characterized by the most severe deprivations.

The following formal model aims to support national planners in determining the types of interventions that should be prioritized and the specific sociodemographic cells, including gender and age, or geographic cells, such as rural/urban, governorate, that should be targeted. Policymakers can also determine the intensity with which a cell should be targeted within a given national context in order to reduce the MPI to a certain level. A bottom-up approach is adopted and considers an existing household-level deprivation matrix and a new target matrix that most efficiently reduces the MPI while expending as few resources as possible. Henceforth, effort refers to a portfolio of resources including fiscal disbursements, manpower, time until impact, and political effort needed to achieve a specific level of reduction in the MPI. For consistency, indicator-specific outlays will be referred to as effort.

Solutions to the problem will be discussed under two formal, integer, linear-optimization models. Those two alternative models have different strengths and weaknesses, make different assumptions, and produce distinct sets of results. This section reviews each models at a high level, while the following sections set out the mathematical formulation used in each model.

The two models can be characterized by the type of information available to the policymaker, or the political and technological restrictions relevant to the targeting of interventions. Model One

assumes that measurement and targeting can be conducted at a household level using complete household-level information, while Model Two uses sociodemographic or geographic information and only uses data on household deprivation in a specific indicator.

**Model One:** The State intervenes at the level of individual households and individual indicators. In other words, the State can change household deprivation status under various indicators and thus the overall MPI. This model is unrealistic given the limited capability of governments to target specific households and ensure that the effort exerted will benefit only households that are deprived and multidimensionally poor. Nevertheless, this model provides an ideal benchmark since it isolates the intervention that has achieved the target MPI with the lowest total effort.

**Model Two:** The State intervenes at the level of sociodemographic or geographic cells, in terms of specific indicators or dimensions. A population cell is any subdivision of households such that every household belongs to only one cell. In this report, the population cell is defined in geographic terms. Unlike the situation in Model One, the State is assumed to be able to restrict its efforts to assisting only deprived households, but cannot distinguish which households are multidimensionally poor. The State is unable to distinguish deprived and multidimensionally-poor households from those that are only indicator deprived. Since the targeting of efforts is imprecise, intervention resources are spread more thinly across all deprived households within the targeted population cell. Alternatively, beneficiary households may be selected from among all deprived households based on imprecise information regarding multidimensional poverty status. As a result, only some of the deprived and multidimensionally poor households will be lifted out of deprivation and multidimensional poverty.

Model Two provides a realistic scenario where uncertainty or limited information prevails. Intervention at the national level is an extreme scenario where the number of population cells is one (population cell = population). Intervention at the group level for those with a similar profile of deprivations, such as a household level intervention for example, is the other extreme case. A small number of households represent each population unit and the State can take into account the full profile of all deprivations.

In general, a central government or the headquarters of a donor agency will allocate a share of its budget to subnational entities who then distribute those resources to individual households to address specific deprivations. While Models One and Two take the form and the degree of national disaggregation as a given, they guide national and local planners' efficient use of resources to maximize impact on the MPI. It is assumed that an intervention targets only one indicator of the MPI, consistent with the idea that policymakers can focus limited resources on specific social areas. Active indicators are therefore assumed to be independent from one another. For example, an intervention that lifts a household out of deprivation in one area is assumed to leave the deprivations of other indicators unchanged. Thus, a policymaker may want to target the indicators that have the largest influence on the MPI.<sup>1</sup>

	Intervention type	
Active input type	Household level	Population cell level
Indicator level	Model One	Model Two

<sup>1</sup> The indicators are assumed to be fully independent. This assumption is made mainly due to a lack of data about the relationship among indicators, such as how indicator one deprivation for a certain household affects indicator two deprivation for the same household. If a relationship exists and is observed then one can model accordingly. Another case that can potentially be modeled is to assume full dependence inside a dimension (cluster of indicators) and full independence of one dimension from the other. This is not addressed in the present report, however.

## 2. Mathematical formulations

This section outlines input variables, decision variables, objective function and constraints for the two models described.

### A. Input and decision variables – Model One

Input variables are divided into two categories: original and computed variables. Original input variables include: the deprivation matrix and MPI-related assumptions (indicators, households, deprivation and multidimensional poverty thresholds, indicator weights); the reduction target for the MPI; the minimum and maximum efforts that can be exercised per indicator; and the effort required to remove one household from deprivation according to a certain indicator. Computed input variables are additional variables that are computed prior to the optimization routine based on original input variables. Table 1 sets out further information on the variable definitions.

**Table 1. Definition of the input, computed input and internal/external decision variables for Model One**

Input variables	
$I$	Set of households
$J$	Set of individual indicators
$k$	Poverty threshold
$\forall j \in J, w_j$	Weights of the various indicators. The sum of all weights is 1
$\forall j \in J, l_j$	Lower bound on the effort spent per indicator
$\forall j \in J, u_j$	Upper bound on the effort spent per indicator
$\forall j \in J, EpF_j$	Effort required to induce a flip per indicator
$\forall i \in I, \forall j \in J, M_{ij}$	Binary deprivation per household and indicator
$\forall i \in I, HS_i$	Household size per household
$\forall i \in I, HW_i^2$	Statistical weight of household
$MPI_s$	Starting MPI (pre-optimization)
$MPI_r$	Reduction required in MPI
Computed input variables	
$\forall i \in I, \forall j \in J, Mw_{ij}$	Weighted deprivation per household and indicator
$\forall i \in I, P_i$	Binary input variable indicating if a household is originally poor (1) or not (0)
External decision variables	
$\forall i \in I, \forall j \in J, N_{ij}$	Binary decision variable member of the post-optimization deprivation matrix N
$\forall j \in J, E_j$	Effort in the corresponding indicator j
Internal decision variables	
$\forall i \in I, C_i$	Contribution of a household to the post optimization MPI. $C_i$ is a continuous variable with a minimum of zero.

<sup>2</sup> Household statistical weights are assumed to equal one, and the input deprivation matrix M is assumed to be complete and represent the entire population (in other words, there are no sample frame or stratification/representation issues).

Decision variables are organized into two categories: external decision variables are visible to the user and are derived from the general logic of the optimization; internal decision variables are introduced for the sake of facilitating the optimization or transforming logical constraints into linear constraints.

## B. The objective function and constraints in logical form – Model One

Model One aims to minimize the total effort exerted:

$$\min \sum_J E_j \quad (\text{Obj 1})$$

The following constraints encompass the basic requirements for MPI reduction optimization. Deprivations can only be reduced and cannot be increased:

$$\forall i \in I, \forall j \in J, N_{ij} \leq M_{ij} \quad (\text{Con 1})$$

Household contribution to the new MPI is assessed using the weighted sum of its deprivations, the weights, household size and household statistical weight in cases where the household is poor post-intervention. In logical form, this means:

$$\forall i \in I, \sum_J N_{ij} \cdot w_j \geq k \Rightarrow C_i = \sum_J N_{ij} \cdot w_j \cdot HS_i \cdot HW_i \quad (\text{Con 2})$$

A complement to constraint 2 is that the contribution of a household to the new MPI is equal to 0 if the household is not poor post-intervention.

$$\forall i \in I, \sum_J N_{ij} \cdot w_j < k \Rightarrow C_i = 0 \quad (\text{Con 3})$$

The value of effort per indicator is:

$$\forall j \in J, E_j = EpF_j \cdot \sum_I HW_i \cdot (M_{ij} - N_{ij}) \quad (\text{Con 4})$$

subject to the minimum:

$$\forall j \in J, E_j \geq l_j \quad (\text{Con 5})$$

and maximum:

$$\forall j \in J, E_j \leq u_j \quad (\text{Con 6})$$

The post-optimization MPI is the sum of the contributions to the MPI by all households divided by population (statistically weighted):

$$\frac{\sum_I C_i}{\sum_I HS_i \cdot HW_i} \leq MPI_s \cdot (1 - MPI_r) \quad (\text{Con 7})$$

### C. Input and decision variables – Model Two

ModelTwo assumes that effort is exercised at the level of population cells. The input variables and the decision variables are affected and thus additional variables are necessary. Those variables are listed in table 2.

**Table 2. Definition of the additional input and decision variables for Model Two**

Input variables	
$\forall i \in I, \forall j \in J, R_{ij}$	A random number between 0 and 1 to determine whether the corresponding entry in the deprivation matrix will be flipped as a result of the effort exerted.
$D$	Set of population cells
$\forall i \in I, GC_i$	Population cell
$I[d]$	Set of households belonging to a population cell d (computed input)
Decision variables	
$\forall j \in J, \forall d \in D, E_{jd}$	Effort in corresponding indicator j and geographic cell d

Efforts are now computed at the level of population cells and indicators.

### D. The objective function and constraints in logical form – Model Two

The objective function in ModelTwo becomes:

$$\min \sum_J \sum_D E_{jd} \quad (\text{Obj 2})$$

That function is subject to the constraints in Model One and adjusted accordingly. Most notably, constraint 4 is replaced by one computation at the indicator and population cell level:

$$\forall j \in J, \forall d \in D, E_{jd} = EpF_j \cdot \sum_{I[d]} HW_i \cdot (M_{ij} - N_{ij}) \quad (\text{Con 8})$$

and constraints 6 and 7 are replaced as follows:

$$\forall j \in J, \sum_D E_{jd} \geq l_j \quad (\text{Con 9})$$

$$\forall j \in J, \sum_D E_{jd} \leq u_j \quad (\text{Con 10})$$

Additional constraints are added to account for the random impact of efforts  $E_j$  on deprivation in indicator  $j$ . The total number of flips that  $E_{jd}$  induces is  $E_{jd}/EpF_j$  flips in column  $j$  of the deprivation matrix. The probability that household  $i$  has its indicator  $j$  flipped as a result of effort  $E_j$  is:

$$\min \left( \frac{E_{jd}/EpF_j}{\sum_{i' \in I[GC_i]} M_{i'j}}, 1 \right)$$

Accordingly, given the random matrix  $R$ , 3 household  $i$  has its indicator  $j$  flipped as a result of effort  $E_j$  when the following condition holds:

$$R_{ij} \leq \frac{E_{jd}/EpF_j}{\sum_{i' \in I[GC_i]} M_{i'j}}$$

Otherwise, household  $i$  does not experience a flip in its indicator  $j$ , regardless of whether it is multidimensionally poor. In logical form, those conditions are:

$$\forall i \in I, \forall j \in J, R_{ij} \leq \frac{E_{jd}}{EpF_j} \Rightarrow N_{ij} = 0 \quad (\text{Con 11})$$

$$\forall i \in I, \forall j \in J, R_{ij} > \frac{E_{jd}}{EpF_j} \Rightarrow N_{ij} = M_{ij} \quad (\text{Con 12})$$

Those conditions ensure that each household that is experiencing deprivation in a certain indicator area is equally likely to be removed from deprivation as a result of an intervention. For example, effort at flipping two households will, on average, result in two households from among the total deprived households being flipped at random, regardless of whether those households are multidimensionally poor.

## E. The integer linear programme used in Models One and Two

The full optimization system for Model One can be written as:

$$\min \sum_J E_j \quad (\text{Obj 3})$$

This is subject to constraints 1–7, when it is assumed that constraints 2 and 3 will be linearized, as illustrated in the annex to this report. That optimization model is an integer linear programme (ILP), since the objective function and constraints are linear with respect to decision variables and certain decision variables ( $N_{ij}$ ) are integers.

Similarly, the full optimization system for Model Two can be expressed as:

$$\min \sum_J \sum_D E_{jd} \quad (\text{Obj 4})$$

This is subject to constraints 1–3, 5 and 8–12, assuming that constraints 2, 3, 11 and 12 are linearized, as illustrated in the annex. That optimization model is also an ILP.

<sup>3</sup> Each cell in this matrix is a random number generated from a uniform distribution of the interval [0,1].



### 3. Simulation and case study

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Chapter 4 describes the inputs and assumptions used to run the alternative optimization models on a sample dataset.

#### A. Baseline input and assumptions

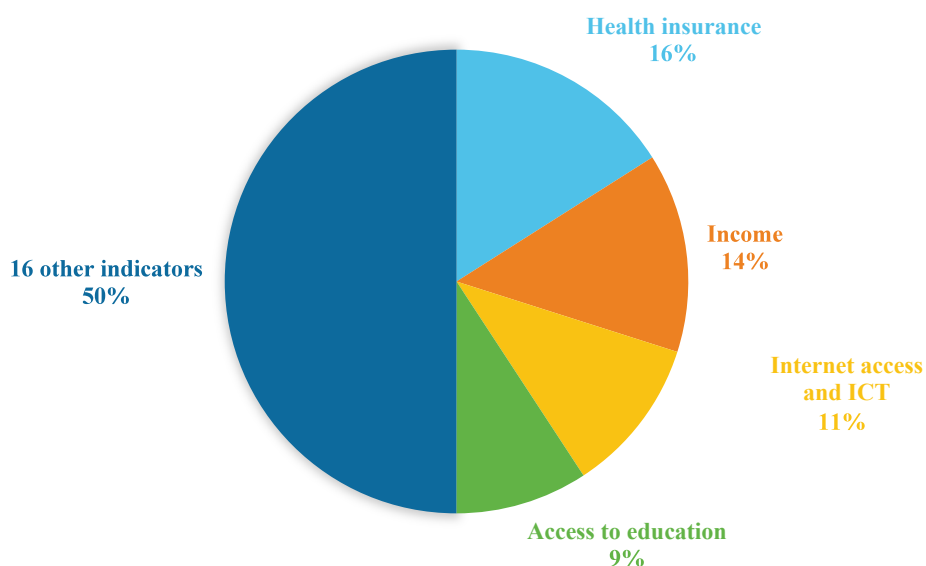
##### 1. Proposed MPI for Lebanon and its normative assumption

The input data used in this example are retrieved from the Labour Force and Household Living Conditions Survey (LFHLCS) for Lebanon 2018–2019. An illustrative deprivation matrix is generated therefrom and model parameters are populated based on a proposed national MPI framework for Lebanon (ESCWA, 2020). The framework’s normative assumptions and results are summarized below.

The MPI framework applied to the LFHLCS produces a binary uncensored deprivation matrix for 38,929 households and 20 indicators. Indicators are equally weighted and belong to one of six equally weighted dimensions. The poverty cut-off is specified at one out of six, or 0.17.

Applying the Alkire Foster method, the results for Lebanon show: the MPI headcount ratio is 0.411; the intensity (or average weighted deprivation of poor people) is 0.273; and the MPI is 0.112. The contribution of each indicator to the MPI is shown in figure 1, and indicators are sorted from most to least influential contributing factor. The four indicators contributing the most are health insurance, income, information and communications technology (ICT) and internet access, and access to education. When combined these indicators contribute to slightly more than 50 per cent of the MPI score.

Figure 1. Indicator contributions to MPI



<sup>4</sup> The survey was conducted by the Central Administration of Statistics, with support from the International Labour Organization and the European Union.

## 2. Optimization inputs (active indicators, cost assumptions, MPI reduction target)

Analysing the full data set in connection with optimization challenges would use up significant computational resources. Therefore, a simpler approach for obtaining adequate results is to repeatedly draw random subsamples, for example 2.5 per cent of the total number of households, apply the same conditions to each sample and aggregate the results. A caveat to this technique is that a poor geographic balance may occur in some of the smaller subsamples, which will affect the results.

Subsamples of 1,000 households were obtained based on the deprivation matrix from the LFHCLS. Each household was characterized by the following properties: typical household size and geographic location, defined in this study as a governorate (1–8).

For Models One and Two, the active indicators subject to optimization must be selected, and so the four indicators with the largest contribution to the MPI were chosen. Flipping all 1 values to 0 in their corresponding columns in the deprivation matrix results in a 79 per cent reduction of the MPI, a disproportionately large change given the 50 per cent contribution of those indicators to the current MPI. Therefore, by focusing solely on those four indicators, the MPI can be reduced by up to 79 per cent from its original value of 0.112.

The selected active indicators each refer to a specific dimension, namely, health, income, ICT and appliances, and education. The associated efforts per flip for the active indicators are set according to the parameters shown in table 3.

**Table 3. Efforts per flip for indicators**

Indicator/dimension	Effort/flip
Health insurance	6
Income	6
Internet access and ICT	3
Access to education	5

The selection of active indicators and assumptions regarding the effort needed to flip them are made for illustration purposes only, and make assumptions regarding the relative costliness of flipping households from a deprived to a non-deprived state for each indicator.

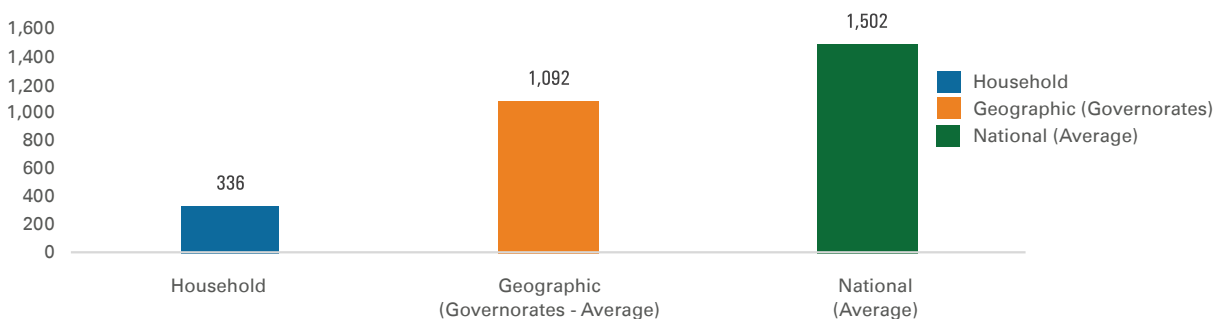
For this exercise, ESCWA has relied on local expertise, subjective expert judgment and prior evidence of changes in deprivation rates (ESCWA, 2021). Furthermore, the target reduction for the MPI was set at 20 per cent of the starting value.<sup>5</sup>

<sup>5</sup> Another approach to measuring the effort each flip would involve using historical State budget allocations over a designated time period with the current year labelled as the base year and the past year denoted as *t-2* and then matching the cost allocations to the changes in the deprivations of indicators or dimensions in the historical household budget surveys and computing the cost per indicator and per flipped household. That assessment could be carried out by dividing the total cost allocated for an indicator over the previous two years by the changes in the deprivation rates over the same period.

## B. Implementation and results

Model One is deterministic by design and can be solved in a single numerical run. For each subsample chosen a new single run must be calculated. In contrast, Model Two can be used to calculate a probabilistic interpretation, given its assumption of limited information or uncertainty. The calculations should be solved repeatedly to confirm the uniqueness and robustness of the results and policy prescriptions. To target at the governorate level, where the population cell = governorate, Model Two is run 100 times, with results averaged across all of the runs. This enables the estimation of average efforts needed to achieve the desired MPI reduction. The case of national targeting where the entire population is included in a single cell is estimated for reference.

**Figure 2. Total units of effort at different intervention levels, Model One.**

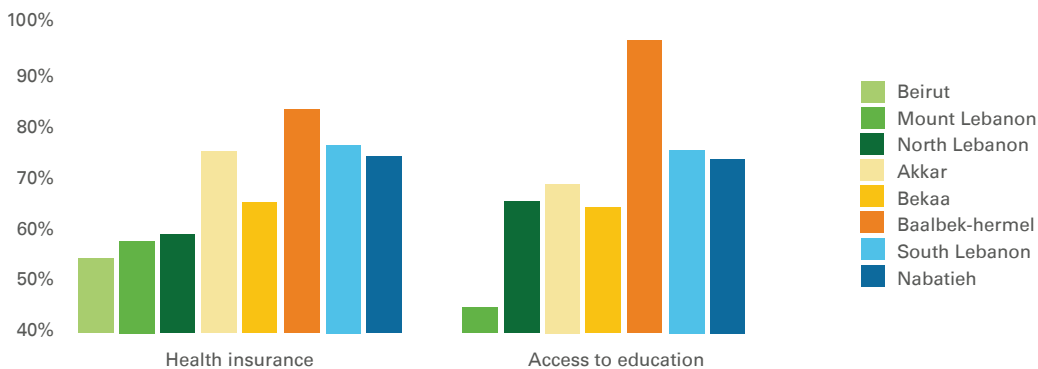


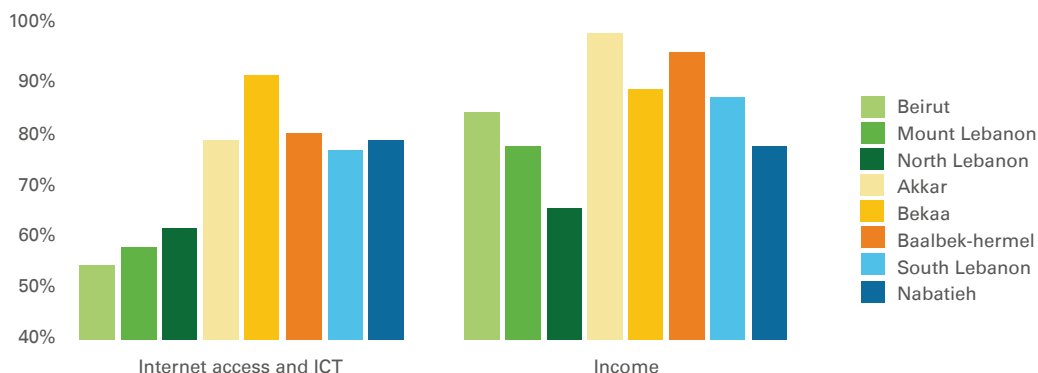
As expected, fully informed household-level optimization yields the lowest total effort to achieve the MPI-reduction target. This is due to a perfect allocation of effort to only deprived and multidimensionally poor households. Limited information targeting at the governorate level also performs better than national targeting. This is because the State can more effectively allocate resources through distribution to different governorates in proportion to their existing multidimensional poverty rates.

Furthermore, figure 3 confirms that the ratio of households that are deprived and poor to all those deprived (Ratio defined as,  $r_{jGC_i} = \frac{\text{Households that are deprived and poor}, j}{\text{Households that are deprived}}$ , where  $J$  is defined as the

set of active individual indicators,  $I$  as the set of households and  $GC_i$  as its geographic cell and  $\forall i \in I, \forall j \in J$  varies across governorates. That ratio is important as it indicates how much of the effort spent on a certain indicator in a population cell contributes to MPI reduction.

**Figure 3. Ratio of deprived and poor to all deprived by indicator and geographic cell**



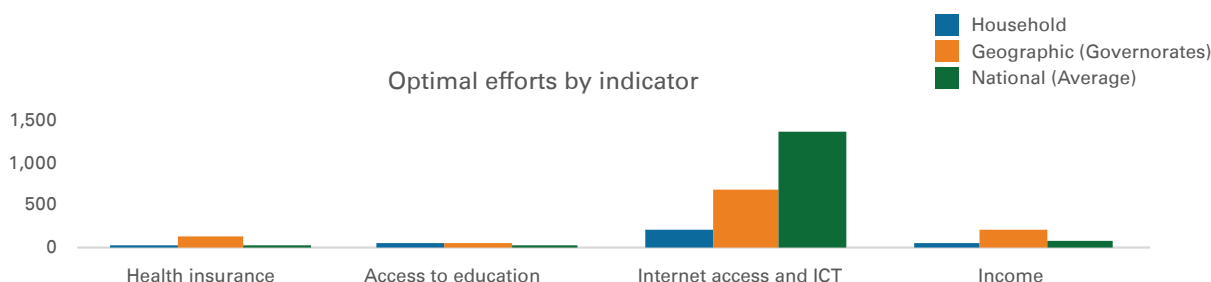


Total effort is unevenly distributed across indicators, as illustrated in figure 4.

All three approaches appear to prioritize reducing deprivations in the Internet access and ICT indicator due to the following:

that indicator has low rates of effort needed per flip ( $E_{pF}$ ) as compared to other indicators; the indicator has a high relative number of deprived households, suggesting that improvements in the indicator could reduce deprivation rates and multidimensional poverty by a large absolute margin; the ratio of those deprived and poor according to the indicator relative to all those deprived is high, meaning that a greater share of those targeted are likely to be lifted out of multidimensional poverty.

**Figure 4. Distribution of total efforts by indicator and intervention level**



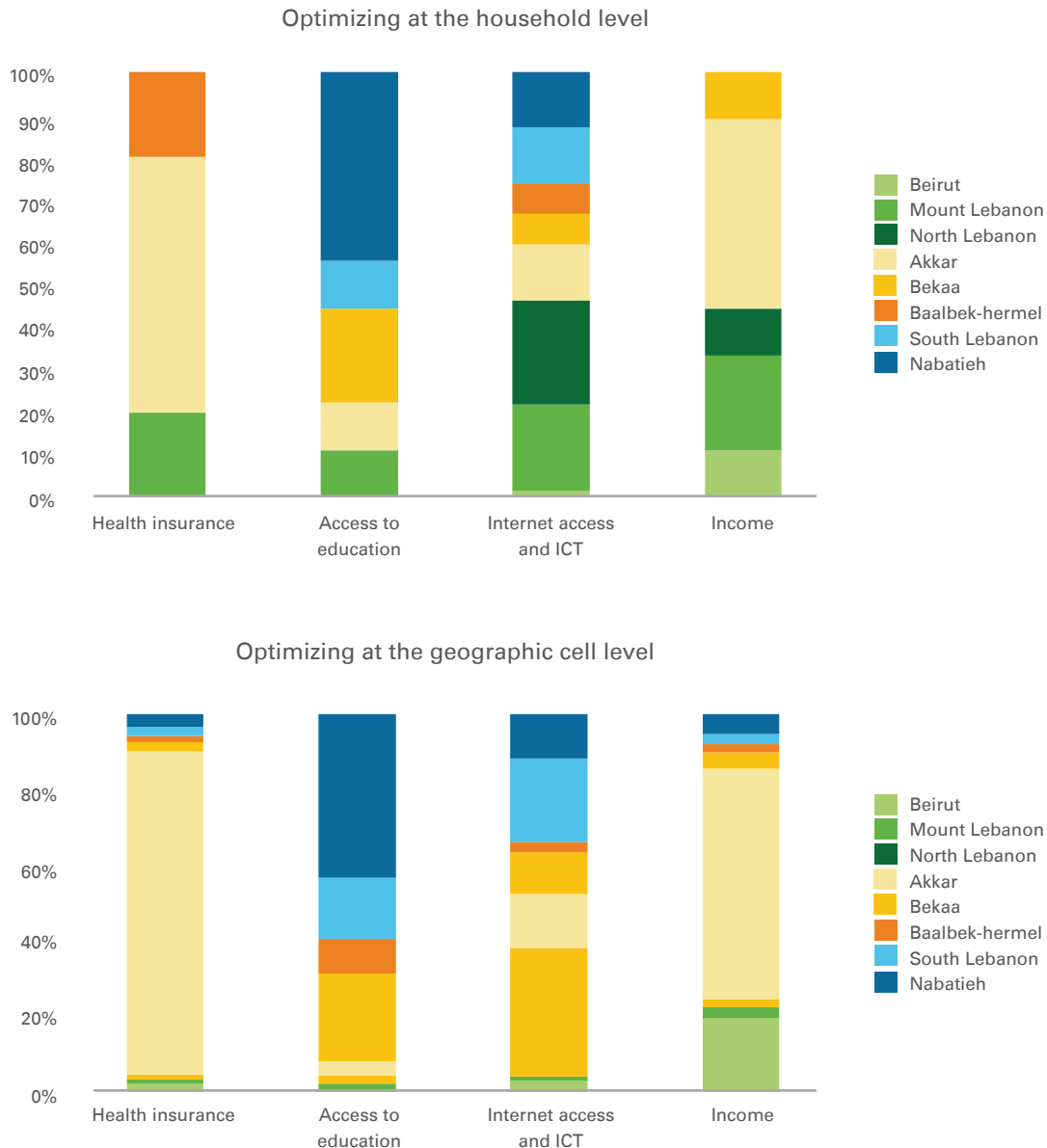
There is a clear difference between household- and governorate-level interventions on the one hand, and national-level interventions on the other.

The national-level approach primarily focuses on the indicator<sup>6</sup> that requires the least effort.

This is not the case for the household- and governorate-level interventions, in which, despite the majority of effort being focused on improving the indicator for Internet access and ICT, considerable effort is also focused on the other indicators.

<sup>6</sup> The authors note that national-level interventions will always focus on a single indicator before moving on to the next one until all deprived households in the first indicator are flipped. The results for the other two approaches do not necessarily focus on all deprived households in a single indicator before they target other indicators.

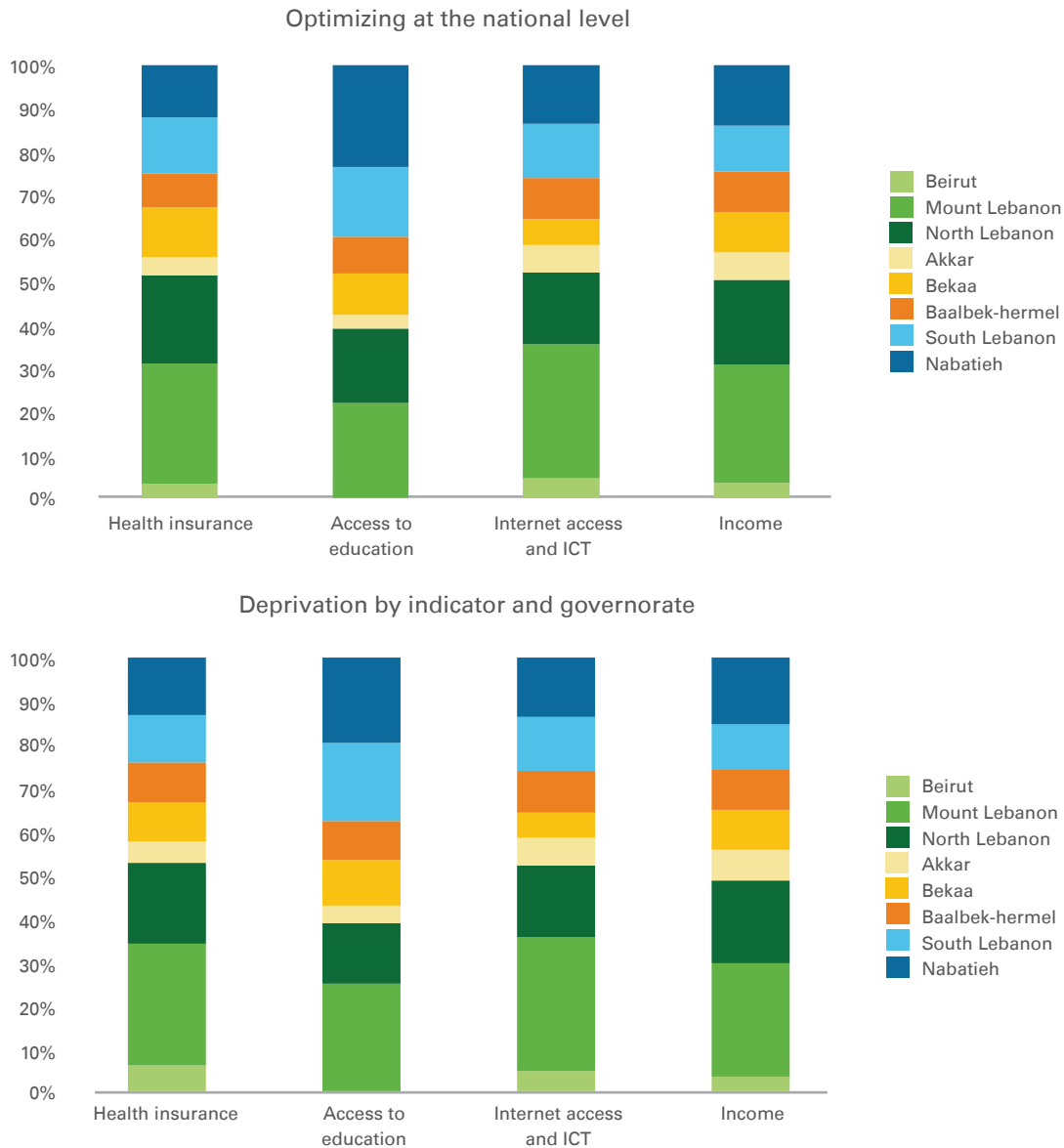
**Figure 5. Distribution of total efforts across all indicators by governorate – Models One and Two**



The distribution of efforts across governorates, as illustrated in figures 5 and 6, indicates clear similarities between Models One and Two. Both Models allocate the greatest amount of effort to the same governorates by indicator: Akkar for health insurance and income, Nabatieh for education, and North Lebanon for Internet access and ICT, while relative allocations to all governorates are similar for access to education and Internet access and ICT indicators.

That consistency is reassuring because it confirms that even limited information targeting at the level of population cells reaches a similar subset of households as does full-information targeting, albeit using more resources.

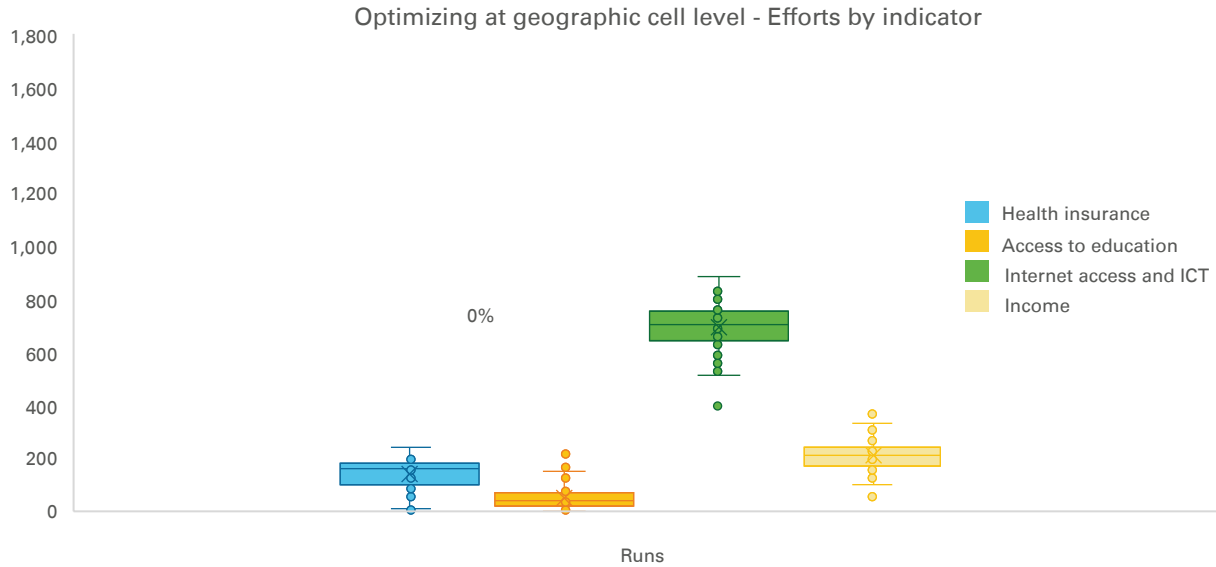
**Figure 6. Distribution of total efforts and resulting deprivations by governorate – national-level approach**



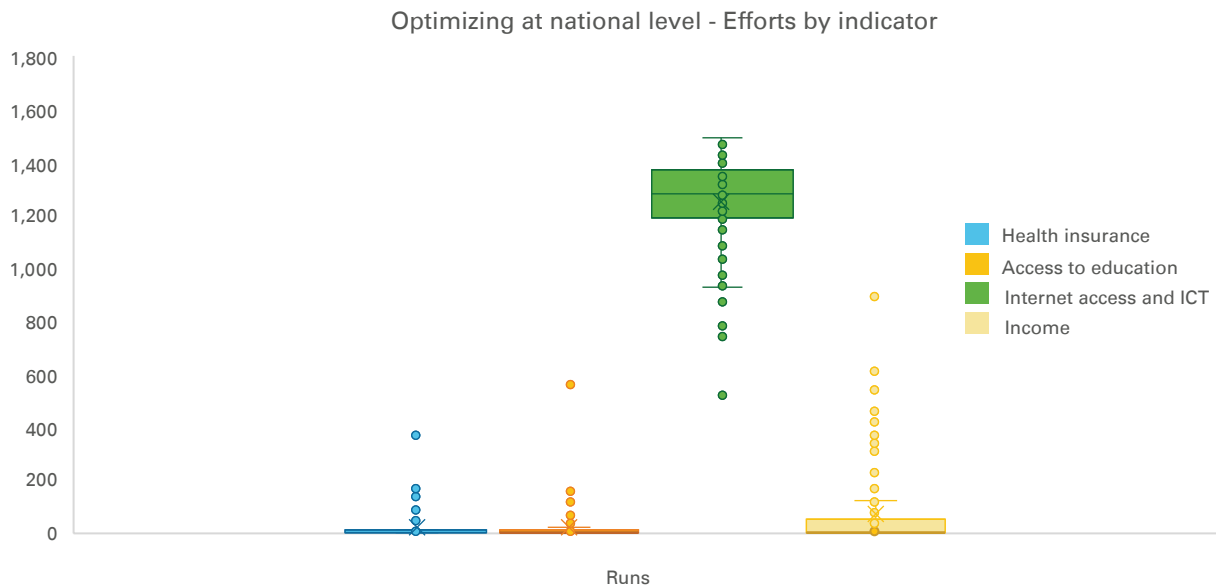
The results at the national level differ from those resulting from the previous two levels and show similar distributions across governorates for all active indicators; the results are in line with the distribution of deprived households by indicator across governorates, specifically, the pre-optimization MPI results disaggregated by governorate. The regional distribution among beneficiaries should therefore follow the distribution of deprived households, namely the uncensored headcount ratio for each indicator disaggregated by governorate.

For the 100 repeated runs of Model Two, differences across all runs were examined. Figure 7 illustrates that estimations of efforts exerted are consistently close to the average values reported in figure 5, with limited spreads above and below the averages across all governorates. In contrast, when compared to the averages shown in figure 6, spreads for the results of national-level targeting, shown in figure 8, are wider.

**Figure 7. Spread of effort across 100 geographic-level runs by indicator**



**Figure 8. Spread of effort across 100 national-level runs by indicator**



Numerous cells and small population size will result in tighter geographic cells. Household-level results can be considered an extreme case in which each household is a unique population cell and all inputs will yield the same result.

## 4. Limitations of the study and potential areas for further analysis

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The two models presented in this report rely on several assumptions and are not without their shortcomings. The following five limitations should be addressed in order to improve analysis.

1. The deprivation matrix is assumed to remain static over the planning horizon.
2. Effort per flip is assumed to be estimable and deterministic.
3. Effort per flip is assumed to be constant across flips achieved.
4. MPI reduction targets are assumed to be a given and remain unchanged over the planning horizon.
5. The active indicators are assumed to be independent of one another.

### A. The deprivation matrix is assumed to remain static over the planning horizon

We assume that the interventions to reduce the MPI are the only factors affecting the deprivation matrix. However, those interventions only create positive change by removing households from deprivation. In reality, other external factors affect the deprivation matrix. Households covered by the deprivation matrix are also assumed to remain there with no accounting for households exiting or new households entering the matrix. Such an assumption might be realistic for short-term planning but may need to be relaxed for longer-term planning. In addition, assumptions also limit which indicators may be made active. For example, indicators where households are stuck in a deprivation status, such as those that have experienced child loss, may not be made active if it is impossible to remove the deprivations affecting them. Improving healthcare access for children will mitigate child mortality, but it is only by preventing additional households from falling into deprivation rather than by lifting presently-deprived households out of poverty that true improvement will be seen. The formulation of the optimization models is based on a fixed deprivation matrix that does not permit a “prevention capability” for current or future households.

To overcome those limitations, the models could be reformulated to include two deprivation matrices: the original deprivation matrix at the time of planning and the forecasted deprivation matrix at the target date in the absence of any additional poverty reduction initiatives. The models should then support target MPI achievement by removing households from deprivation and preventing additional households from falling into deprivation across indicators.

### B. Effort per flip is assumed to be readily estimable and deterministic

Effort per flip estimates are necessary parameters for the models and future investigations will consider formalizing how they can be estimated. One approach is to map budget spending per dimension/indicator of MPI over a certain period against the observed change in the deprivation matrix over the same period. That calculation would provide an estimate of the financial effort needed to achieve a flip per dimension/indicator.



### **C. Effort per flip is assumed to be constant across flips achieved**

The effort required to flip one additional household is assumed to be constant in the study. While that might be realistic for specific types of indicators or for limited ranges of flips for an indicator, it is not necessarily correct across the board. A generic representation of effort per flip should therefore be considered. The main difficulty in achieving this is computational, since assuming non-constant values requires more advanced, non-linear optimization models. Those models are more computationally demanding and may not be feasible.

Future studies should aim to develop more realistic functional forms for effort per flip and approximate them using constant functions in order to keep the optimization models linear, reduce their computational complexity and avoid having several competing solutions.

### **D. MPI reduction targets are assumed to be a given and remain unchanged over the planning horizon**

The MPI reduction target is an input parameter for the two models. Given allocated budgets for poverty reduction, one could run the models with different MPI reduction targets and assess whether the targets are achievable (model output results in a feasible solution) or non-achievable (model output results in an unfeasible solution).

### **E. The active indicators are assumed to be independent of one another**

Policymaker interventions for specific active indicators are assumed to not affect deprivation in other indicators. That result is restrictive, as it is assumed that changes in active indicators do not distort inactive indicators. Thus, assessing possible interdependence across indicators is a challenge with the two models. One approach to mitigate that problem is to target dimensions of the MPI rather than individual indicators, since dimensions are less dependent on one another than indicators. Moreover, State interventions may have a greater impact on dimension-level deprivation reduction than on indicator-level reductions.<sup>7</sup>

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<sup>7</sup> A preliminary formulation of dimension-level optimization models and their results are available on request.

## 5. Conclusion

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This paper is an initial attempt at supporting national planners in determining the types and magnitude of interventions across population cells and the specific population cells that should be targeted within a national context to efficiently achieve MPI reduction. The investigation used a bottom-up approach by examining an existing deprivation matrix and generating a new target matrix that achieves a desired reduction in the MPI with minimal effort.

Model One sheds light on challenges to be addressed and provides lower limits on the level of effort needed to induce the required MPI reduction. ModelTwo is a stochastic model where the initial restrictive assumption of ultimate State targeting capability is relaxed. The State intervenes to alleviate deprivation in an impartial and limited manner, regardless of the multidimensional poverty status of the recipients. The beneficiaries of a specific State intervention are assumed to be randomly selected from the pool of deprived households in the relevant indicator. The State can either undertake its interventions at the national level or at specific geographic cells in order to improve its targeting capabilities.

The two models were characterized, parameterized and tested against sample data from Lebanon. The study highlights how the results can support decision makers by highlighting the interventions that are effective in reducing the MPI and by identifying the geographic cells that should be prioritized.

Interventions at a geographic cell level are preferred to those implemented at the national level since those offer policymakers the flexibility of being guided either by household-level results or by national-level results. Intervention at the household level enables the setting of lower limits on total efforts exerted and to measure the efficiency of the more realistic ModelTwo.

A policymaker might be interested in computing the savings generated from a deeper geographic cell targeting (Qaza-level) rather at the higher level of governorate-level geographic cells, because this can inform whether the savings justify additional bureaucratic requirements needed for decentralization. ModelTwo can be run according to two scenarios: governorates are treated as demographic cells, or the entire country is treated as a single population cell in which governorates with different multidimensional-poverty rates are not treated differently.

Modelling assesses the lowest cost per flip, the high ratio of deprived and poor versus deprived groups, as well as high deprivation pockets among the population. In conclusion, in light of the knowledge gained regarding the relative effort required for various indicators, the optimization models presented here should guide policymakers regarding which indicators and regions should be focused on to achieve target reductions in MPI.

# Annex

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The linear equivalent for some of the constraints shall be derived. We note the following equivalence:

$$A \Rightarrow B \equiv B \vee \neg A$$

Therefore enforcing  $A \Rightarrow B$  is equivalent to enforcing  $B \vee \neg A$ . The latter is enforced if at least one of the two sides of the “or” relation is imposed.

Starting with Model One, constraints 2 and 3 are displayed in logical form. Constraint 2 is equivalent to:

$$\forall i \in I, \left( C_i = \sum_J N_{ij} \cdot w_j \cdot HS_i \cdot HW_i \right) \vee \left( \sum_J N_{ij} \cdot w_j < k \right) \quad (\text{Con 13})$$

(Con 13) is equivalent to the following three linear constraints where  $b1_i$  are binary decision variables and bigM is a sufficiently large number:

$$\forall i \in I, C_i + \text{bigM} \cdot b1_i \geq \sum_J N_{ij} \cdot w_j \cdot HS_i \cdot HW_i \quad (\text{Lin 1})$$

$$\forall i \in I, C_i - \text{bigM} \cdot b1_i \leq \sum_J N_{ij} \cdot w_j \cdot HS_i \cdot HW_i \quad (\text{Lin 2})$$

$$\forall i \in I, \sum_J N_{ij} \cdot w_j - (1 - b1_i) \cdot \text{bigM} < k \quad (\text{Lin3})$$

and where  $b1_i$  are binary decision variables required to transform logical constraints into linear constraints.

The logic behind this equivalence is the following: When  $b1_i = 1$ , (Lin 1) and (Lin 2) are imposed with a neutralized effect of bigM and (Lin 3) is always true. This equivalently imposes the first element of the “or” relation in (Con 13) while relaxing the second element.

When  $b1_i = 0$ , (Lin 1) and (Lin 2) are always true and (Lin 3) is imposed with a neutralized effect of bigM. This equivalently relaxes the first element of the “or” relation in (Con 13) and imposes the second element.

Constraint 3 is equivalent to:

$$\forall i \in I, (C_i = 0) \vee \left( \sum_J N_{ij} \cdot w_j \geq k \right) \quad (\text{Con 14})$$

(Con 14) is equivalent to the following two linear constraints where  $b2_i$  are binary decision variables and bigM is a sufficiently large number:

$$\forall i \in I, C_i - \text{bigM} \cdot b2_i \leq 0 \quad (\text{Lin 4})$$

$$\forall i \in I, \sum_J N_{ij} \cdot w_j + \text{bigM} \cdot (1 - b2_i) \geq k \quad (\text{Lin 5})$$

and where  $b2_i$  are binary decision variables required to transform logical constraints into linear constraints.

The logic behind this equivalence is the following:

When  $b2_i = 0$ , (Lin 4) is imposed with a neutralized effect of big M while (Lin 5) is always true. This equivalently enforces the first element in the “or” relation in (Con 14) and relaxes the second element. In fact, this imposes  $C_i \leq 0$ , but given that  $C_i$  is defined as a continuous decision variable with a minimum of 0, then this imposes that  $C_i = 0$ . When  $b2_i = 1$ , (Lin 5) is imposed with a neutralized effect of big M while (Lin 4) is always true. This equivalently enforces the second element in the “or” relation in (Con 14) and relaxes the first element.

Looking at the linear representations of constraints 2 and 3, identified above as (lin 1 to 5), one can notice that  $b2_i$  can be replaced by  $(1 - b1_j)$  to reduce the number of decision variables.

For ModelTwo, in addition to constraints 2 and 3, which are linear equivalents, constraints 11 and 12 must be linearized as follows. Constraint 11 can be written as:

$$\forall i \in I, \forall j \in J, (N_{ij} = 0) \vee \left( R_{ij} > \frac{\frac{E_j}{EpF_j}}{\sum_{i' \in I[Gc_i]} M_{i'j}} \right) \quad (\text{Con 15})$$

(Con 15) is equivalent to the following two linear constraints where  $b2_{ij}$  are binary decision variables:

$$\forall i \in I, \forall j \in J, N_{ij} - \text{bigM} \cdot b2_{ij} \leq 0 \quad (\text{Lin 6})$$

$$\forall i \in I, \forall j \in J, \frac{E_j}{EpF_j \sum_{i' \in I[Gc_i]} M_{i'j}} - \text{bigM} \cdot (1 - b2_{ij}) < R_{ij} \quad (\text{Lin 7})$$

(Con 12) can be written as:

$$\forall i \in I, \forall j \in J, (N_{ij} = M_{ij}) \vee \left( R_{ij} \leq \frac{\frac{E_j}{EpF_j}}{\sum_{i' \in I[Gc_i]} M_{i'j}} \right) \quad (\text{Con 16})$$

(Con 16) is equivalent to the following three linear constraints where  $b3_{ij}$  are binary decision variables and bigM is a sufficiently large number:

$$\forall i \in I, \forall j \in J, N_{ij} + \text{bigM} \cdot b3_{ij} \geq M_{ij} \quad (\text{Lin 8})$$

$$\forall i \in I, \forall j \in J, N_{ij} - \text{bigM} \cdot b3_{ij} \leq M_{ij} \quad (\text{Lin 9})$$

$$\forall i \in I, \forall j \in J, \frac{E_j}{EpF_j \sum_{i' \in I[Gc_i]} M_{i'j}} + \text{bigM} \cdot (1 - b3_{ij}) \geq R_{ij} \quad (\text{Lin 10})$$

Looking at the linear representations of constraints 11 and 12, identified above as (lin 6 to 10), one can notice that  $b3_{ij}$  can be replaced by  $(1 - b2_{ij})$  to reduce the number of decision variables.

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Strengthening  
Social Protection for  
Pandemic Responses  
**Building social  
protection capacities**



Strengthening  
Social Protection for  
Pandemic Responses  
**Guiding poverty  
reduction**



Strengthening  
Social Protection for  
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**Advancing care  
economy**



In the post COVID-19 era, survey data and the Multidimensional Poverty Index (MPI) can be used to assess the impact of COVID-19 on multidimensional poverty. The present report provides guidance to policymakers with a view to ensuring efficient resource allocation, promoting recovery from COVID-19 and achieving the Sustainable Development Goals. The study supports national planners in determining effective types of interventions and their scope, as well as best practices for targeting specific population groups, and efficiently reducing MPI factors. Survey data from Lebanon confirms that governorate-level interventions facilitate efficient resource allocation across population groups and MPI indicators.

